

**Đề bài**

a)  $\int_0^{\frac{\pi}{24}} \tan\left(\frac{\pi}{3} - 4x\right) dx$  (đặt  $u = \cos\left(\frac{\pi}{3} - 4x\right)$ )

b)  $\int_{\frac{\sqrt{3}}{5}}^{\frac{3}{5}} \frac{dx}{9+25x^2}$  (đặt  $x = \frac{3}{5} \tan t$ )

c)  $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$  (đặt  $u = \cos x$ )

d)  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sqrt{1+\tan x}}{\cos^2 x} dx$  (đặt  $u = \sqrt{1+\tan x}$ )

**Hướng dẫn giải**

a) Đặt  $u = \cos\left(\frac{\pi}{3} - 4x\right)$

b) Đặt  $x = \frac{3}{5} \tan t$

c) Đặt  $u = \cos x$

d) Đặt  $u = \sqrt{1+\tan x}$

Đáp án bài 12 trang 147 sgk giải tích lớp 12

a) Ta có:  $I = \int_0^{\frac{\pi}{24}} \tan\left(\frac{\pi}{3} - 4x\right) dx = \int_0^{\frac{\pi}{24}} \frac{\sin\left(\frac{\pi}{3} - 4x\right)}{\cos\left(\frac{\pi}{3} - 4x\right)} dx$

Đặt  $u = \cos\left(\frac{\pi}{3} - 4x\right) \Leftrightarrow du = 4 \sin\left(\frac{\pi}{3} - 4x\right) dx$ .

Đổi cận:  $\begin{cases} x = 0 \Rightarrow u = \frac{1}{2} \\ x = \frac{\pi}{24} \Rightarrow u = \frac{\sqrt{3}}{2} \end{cases}$

Khi đó:  $I = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{du}{4u} = \frac{1}{4} \ln|u| \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \frac{1}{4} \left( \ln \frac{\sqrt{3}}{2} - \ln \frac{1}{2} \right) = \frac{1}{4} \ln \sqrt{3}$

b) Đặt  $x = \frac{3}{5} \tan t \Leftrightarrow dx = \frac{3}{5 \cos^2 t} dt = \frac{3}{5} (\tan^2 t + 1) dt$ .

Đổi cận:  $\begin{cases} x = \frac{\sqrt{3}}{5} \Rightarrow t = \frac{\pi}{6} \\ x = \frac{3}{5} \Rightarrow t = \frac{\pi}{4} \end{cases}$

$I = \int_{\frac{\sqrt{3}}{5}}^{\frac{3}{5}} \frac{dx}{9+25x^2} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{3(\tan^2 t + 1) dt}{5(9+25 \cdot \frac{9}{25} \tan^2 t)}$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{3(\tan^2 t + 1)}{5 \cdot 9(\tan^2 t + 1)} dt = \frac{1}{15} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} dt = \frac{t}{15} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{\pi}{180}$

$$\text{c) Ta có: } I = \int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx = \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \cos^4 x \sin x dx$$

$$\text{Đặt } u = \cos x \Rightarrow du = -\sin x dx$$

$$\text{Đổi cận: } \begin{cases} x = 0 \Leftrightarrow u = 1 \\ x = \frac{\pi}{2} \Rightarrow u = 0 \end{cases}$$

$$\Rightarrow I = - \int_1^0 (1 - u^2) u^4 du = \int_0^1 (u^4 - u^6) du$$

$$I = \left( \frac{u^5}{5} - \frac{u^7}{7} \right) \Big|_0^1 = \frac{2}{35}$$

$$\text{d) Đặt } u = \sqrt{1 + \tan x} \Leftrightarrow u^2 = 1 + \tan x \Leftrightarrow 2u du = \frac{1}{\cos^2 x} dx$$

$$\text{Đổi cận: } \begin{cases} x = -\frac{\pi}{4} \Rightarrow u = 0 \\ x = \frac{\pi}{4} \Rightarrow u = \sqrt{2} \end{cases}$$

$$\Rightarrow I = \int_0^{\sqrt{2}} u \cdot 2u du = 2 \int_0^{\sqrt{2}} u^2 du = 2 \frac{u^3}{3} \Big|_0^{\sqrt{2}} = \frac{2}{3} \cdot 2\sqrt{2} = \frac{4\sqrt{2}}{3}$$