

Giải bài 9 trang 161 sgk toán Đại Số lớp 10

Đề bài:

Tính giá trị biểu thức

a) $4(\cos 24^{\circ} + \cos 48^{\circ} - \cos 84^{\circ} - \cos 12^{\circ})$

b) $96\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6}$

c) $\tan 9^{\circ} - \tan 63^{\circ} + \tan 81^{\circ} - \tan 27^{\circ}$

Đáp án:

$$\begin{aligned} & \cos 24^{\circ} + \cos 48^{\circ} \\ &= \cos(36^{\circ} - 12^{\circ}) + \cos(36^{\circ} + 12^{\circ}) \\ &= 2 \cos 36^{\circ} \cos 12^{\circ} \\ \cos 84^{\circ} + \cos 12^{\circ} &= 2 \cos 36^{\circ} \cos 48^{\circ} \\ 4(\cos 24^{\circ} + \cos 48^{\circ} - \cos 84^{\circ} - \cos 12^{\circ}) \\ &= 8 \cos 36^{\circ} (\cos 12^{\circ} - \cos 48^{\circ}) \\ &= 8 \cos 36^{\circ} \cdot 2 \sin 30^{\circ} \cdot \sin 18^{\circ} \\ &= 8 \cos 36^{\circ} \sin 18^{\circ} \\ &= 8 \cos 36^{\circ} \cdot \sqrt{\frac{1 - \cos 36^{\circ}}{2}} \end{aligned}$$

Đặt $36^\circ = x$ ta có:

$$\sin 3x = \sin (180^\circ - 3x) = \sin 2x$$

$$\Leftrightarrow 3 \sin x - 4 \sin^3 x = 2 \sin x \cos x$$

$$\Leftrightarrow 3 - 4(1 - \cos^2 x) = 2 \cos x$$

$$\Leftrightarrow 4 \cos^2 x - 2 \cos x - 1 = 0$$

$$\Rightarrow \cos x = \cos 36^\circ = \frac{1 + \sqrt{5}}{4}$$

$$\text{Vậy: } 4(\cos 24^\circ + \cos 48^\circ - \cos 84^\circ - \cos 12^\circ) = 2(1 + \sqrt{5}) \sqrt{\frac{3 - \sqrt{5}}{8}} = 2$$

$$\begin{aligned} \text{b) } & 96\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} \\ &= 48\sqrt{3} \sin \frac{\pi}{24} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} \\ &= 24\sqrt{3} \sin \frac{\pi}{12} \cos \frac{\pi}{12} \cos \frac{\pi}{6} \\ &= 12\sqrt{3} \sin \frac{\pi}{6} \cos \frac{\pi}{6} = 6\sqrt{3} \sin \frac{\pi}{3} = 9 \end{aligned}$$

$$\begin{aligned} \text{c) } & \tan 9^\circ - \tan 63^\circ + \tan 81^\circ - \tan 27^\circ \\ &= \frac{\cos 81^\circ}{\sin 81^\circ} + \frac{\sin 81^\circ}{\cos 81^\circ} - \left(\frac{\cos 27^\circ}{\sin 27^\circ} + \frac{\sin 27^\circ}{\cos 27^\circ} \right) \\ &= \frac{1}{\sin 81^\circ \cdot \cos 81^\circ} - \frac{1}{\sin 27^\circ \cdot \cos 27^\circ} \\ &= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2}{\cos 72^\circ} - \frac{2}{\cos 36^\circ} \\ &= \frac{2}{2\cos^2 36^\circ - 1} - \frac{2}{\cos 36^\circ} \end{aligned}$$

Thay $\cos 36^\circ = \frac{1 + \sqrt{5}}{4}$ ta được: $\tan 9^\circ - \tan 63^\circ + \tan 81^\circ - \tan 27^\circ = 4$